

# **Workshop on Numerical Methods**

# for Challenging Problems







### **Tianyuan Mathematical Center in Southeast China**

Tianyuan Mathematical Center in Southeast China (TMSE) is one of the five national mathematical centers approved and supported by the Tianyuan Mathematics Fund of the National Natural Science Foundation of China. The unveiling ceremony of the Center was held on January 8th 2019, which marks its official launch and operation.

Tianyuan Mathematics Fund was set up in 1990 with the aim of building China into a strong country in mathematics. It is in 2017 that the academic leadership committee of the Tianyuan Mathematics Fund of the National Natural Science Foundation of China launched the programme of establishing Tianyuan Mathematical Centers for balanced regional development of mathematics. The programme focuses on providing platforms for collaboration and research, and aims to enhance the research in relevant fields, foster research strengths and promote the progress of mathematical sciences. So far there are five Tianyuan Mathematical Centers which are respectively located in the southwest, northwest, northeast, southeast and central of China.

TMSE is based at Xiamen University and co-supported by several other universities in Fujian Province, Zhejiang Province, Guangdong Province, Jiangxi Province and Hainan Province. Centering upon the research on pure mathematics and its interdisciplinary application, TMSE will organize a variety of academic activities with a view to pooling high-caliber talents, promoting international cooperation, cultivating young mathematicians and ultimately advance the progress of mathematical sciences in the southeast part of China.

Under the guidance of the academic leadership committee of the Tianyuan Mathematics Fund of the National Natural Science Foundation of China and with the great support of Xiamen University and the joint efforts of its partner universities for TMSE, the center will make great strides in fostering first-class mathematical talents, producing world-class research and developing into a world-renowned platform for talent cultivation, joint research and academic cooperation.

## **Tianyuan Mathematical Center in Southeast China**

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## Information

### **Organizers:**

Mejdi Azaïez, Bordeaux INP Faker Ben Belgacem, UT Compiègne Huangxin Chen, Xiamen University Jie Shen, Eastern Institute of Technology, Ningbo Chuanju Xu, Xiamen University

### **Program Coordinator:**

Mengjie Liu (86-592-2580036)

### **Conference Venue:**

Conference Room S103 at Experiment Building, Haiyun Campus at

Xiamen University (in Chinese: 厦门大学海韵园实验楼 S103)

### Accommodation:

### CURIO COLLECTION by Hilton

(in Chinese: 厦门希尔顿格芮精选酒店)

No.6-8 Longhushan Road, Zengcuoan, Siming District, Xiamen, 361005,

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## Schedule

November 12 <sup>th</sup> , 2024		
All Day	Check-in & Registration	Hilton
18:30	Dinner	питоп

November 13 <sup>th</sup> , 2024 (Conference Room S103 at Experiment Building)		Chair	
08:30-08:45	<b>Opening Ceremony</b>		
08:45-09:20	<b>Jie Shen</b> EIT	Stable higher-order decoupled schemes for Navier-Stokes equations	
09:20-09:55	Frédéric Hecht Sorbonne University	Calculation of the localisation landscape of the stationary waves and of the level of the percolation with FeeFem++	Chuanju Xu
09:55-10:30	<b>Yongyong Cai</b> Beijing Normal University	Improved error estimates of the splitting methods for the long time dynamics for the weakly nonlinear Schrödinger equations	
10:30-10:50	Tea Break		
10:50-11:25	<b>Tao Zhou</b> AMSS, CAS	Derivative-free deep learning method for parabolic PDEs and HJB equations	Mejdi
11:25-12:00	Zakaria Belhechmi UHA	Motion analysis: optimal transport point of view	Azaïez
12:00-13:30	Lunch (at Dafengyuan)		
13:30-14:15	Hongxing Rui Shandong University	Discrete Fracture Matrix Approach based with Petrov-Galerkin IFE for fractured porous media flow	Tao Zhou

14:15-14:50	<b>Mohamed Fahmi Ben Hassen</b> IAU	Blow-up result for a weakly coupled system of two euler-poisson-darboux-tricomi equations with time derivative nonlinearity	
14:50-15:25	<b>Rongliang Chen</b> SIAT, CAS	ParallelScalableDomainDecompositionMethodsforPersonalizedBiomechanicsSimulationsandClinicalApplications	
15:25-15:45	Tea Break		
15:45-16:20	Sidi-Mahmoud Kaber Sorbonne University	A Parallel Exponential Integrator Scheme for Linear Differential Equation	
16:20-16:55	<b>Jiang Yang</b> SUSTech	A post-processing method for \$Q\$-tensor dynamics preserving the physical range	Zakaria Belhechmi
16:55-17:30	Erwan Liberge La Rochelle University	Lattice Boltzmann for modeling fluid-structure interaction	
17:40	Dinner (at Hilton)		

(0	November 14th, 2024 (Conference Room S103 at Experiment Building)Chair		
08:30-09:05	<b>Mejdi Azaïez</b> Bordeaux INP	Least-squares pressure recovery in rom for incompressible flows	
09:05-09:40	Weiying Zheng AMSS, CAS	Perfectly matched layer method for the wave scattering problem by a step-like surface	Jie Shen
09:40-10:15	<b>Cyril Alléry</b> La Rochelle University	Optimal flow control using reduced-order models	
10:15-10:35	Tea Break		
10:35-11:10	Xiaoping Xie Sichuan University	An energy-stable mixed finite element method for Rosensweig ferrofluid flow model	Sidi-
11:10-11:45	Fouzia Achchaq Bordeaux University	Recent advancements and locks on the development and use of heat storage materials for mid- and high temperature applications	Mahmoud Kaber
12:00-13:30	Lunch (at Dafengyuan)		
13:30-18:00	Free discussion		
18:30		Banquet (at City Hotel Xiamen)	

(0	November 15 <sup>th</sup> , 2024 (Conference Room S103 at Experiment Building)		
08:45-09:20	<b>Liwei Xu</b> UESTC	Numerical solutions of wave scattering problems: BIEM,FEM and Coupling	
09:20-09:55	<b>Eya Bejaoui</b> UT Compiègne	Optimization Algorithms for Thermal Energy Storage System	Hongxing Rui
09:55-10:30	Kailiang Wu SUSTech	A Journey into Bound-Preserving Schemes and Theory	
10:30-10:50	Tea Break		
10:50-11:25	<b>Carla Cornillon</b> I2M , Bordeaux University	Use of Clinical Data for the Constuction of Simplified Personalized FE Models in the Context of Pressure Ulcer Prevention	Erwan Liberge
11:25-12:00	<b>Zhenlin Guo</b> CSRC	Phase-field method for multi-component vesicle in fluids	Liberge
12:00-13:30	Lunch (at Dafengyuan)		
13:30-14:15	<b>Xianmin Xu</b> AMSS, CAS	Applications of the Onsager variational principle in numerical analysis	
14:15-14:50	<b>Jérôme Jansen</b> Bordeaux INP	A projection based solver for subsonic compressible flows, applications to natural convection problems	Liwei Xu
14:50-15:25	<b>Shixin Xu</b> Duke Kunshan University	Multidomain Model for Optic Nerve Microcirculation: Roles of Glial Cells and Perivascular Spaces	
15:25-15:45	Tea Break		
15:45-16:20	<b>Shuyu Sun</b> KAUST	Efficient and Structure-Preserving Algorithms for Gradient Flow Formulation of the Kohn-Sham Density Functional Theory	Huangxin Chen

16:20-16:55	<b>Xiaoli Li</b> Shandong University	A class of higher-order length preserving and energy decreasing IMEX schemes for the Landau-Lifshitz equation
17:30	Dinner (at Dafengyuan)	

## **Title and Abstracts**

## Recent advancements and locks on the development and use of heat storage materials for mid- and high temperature applications

Fouzia Achchaq (Bordeaux University)

In this work, we will first present the acquired knowledge about the use of molten salts as heat storage materials for high-temperature applications, such as concentrated solar power plants (CSPs). This knowledge is based on the literature available from the 1970s to the present day. After summarising the main results obtained and the advances made, we will highlight the remaining locks, their causes and the subsequent direction given to this area of research. Then, in the second part, we will introduce a new family of promising candidates for the same kind of applications: the peritectic compounds. A comparison with the eutectic ones mainly used in CSPs' thermal energy storage systems is done, and the first experimental results related to the development of the most promising of them, Li4Br(OH)3 belonging to the salt family, will be presented. Eventually, this presentation will end with the results obtained on the exploitation of sensible and latent heats of another salt. Its characterisation revealed unexpected properties for a rapid development of ultra-compact heat storage systems to be integrated in already existing power plants, as demonstrated by the comparison with Solar Salt, the most studied molten salt-based heat storage material that is also already used in marketed CSPs. New developments regarding PCMs for mid-temperature applications will be also briefly presented.

### Optimal flow control using reduced-order models

Cyril Alléry (La Rochelle University)

Solving optimal control problems using genetic algorithms or gradient descent methods based on adjoint equations requires significant computational time and storage capacity, which limits their use, especially in fluid mechanics. To overcome these issues, it is possible to use intrusive or non-intrusive model reduction techniques based on Proper Orthogonal Decomposition (POD). The intrusive approaches consist in constructing a system of small differential equations, which constitutes the reduced model, obtained by Galerkin projection of the equations onto each element of the POD spatial basis. However, it is known that POD bases are only valid for optimization parameters close to those for which they were constructed. To overcome this difficulty, the reduced optimal control algorithm, commonly used in the literature, is enriched with an intermediate step of POD basis adaptation. This enrichment approach consists of adapting the reduced basis for a new value of the control parameter by interpolation on a set of POD bases previously calculated for a range of control parameters. To achieve this, an interpolation technique based on the properties of the tangent subspace of the Grassmann manifold (ITSGM) is considered. On the other hand, non-intrusive approaches, require no prior knowledge of the model's equations. The parameterized flow data are used in their reduced form obtained by POD (Proper Orthogonal Decomposition), and solutions prediction is made by interpolating the temporal and spatial POD subspaces through ITSGM or Riemannian barycentric interpolation. The robustness of reduced optimization algorithms (based on intrusive or non-intrusive reduced approaches) in terms of computational time and accuracy is demonstrated on optimal control problems such as theflow around a cylinder, the flow in a driven cavity, or the turbulent mixed-convection flow in a cavity.

#### Least-squares pressure recovery in rom for incompressible flows

#### Mejdi Azaïez (Bordeaux INP)

In this work, we introduce a method to recover the reduced pressure for Reduced Order Models (ROMs) of incompressible flows. The pressure is obtained as the least-squares minimum of the residual of the reduced velocity with respect to a dual norm. We prove that this procedure provides a unique solution whenever the full-order pair of velocity-pressure spaces is inf-sup stable. We also prove that the proposed method is equivalent to solving the reduced mixed problem with reduced velocity basis enriched with the supremizers of the reduced pressure gradients. Optimal error estimates for the reduced pressure are obtained for general incompressible flow equations and specifically, for the transient Navier-Stokes equations. We also perform some numerical tests for the flow past a cylinder and the lid-driven cavity flow which confirm the theoretical expectations, and show an improved convergence with respect to other pressure recovery methods.

### **Optimization Algorithms for Thermal Energy Storage System**

Eya Bejaoui (UT Compiègne)

Thermal Energy Storage (TES) systems are expected to have high energy storage capacities. However, their poor thermal conductivities are an important drawback. The main effect is a slow heat flowing across the medium: the contrasting heat storage rates may show up. Encapsulate a high conductive material in the TES support is an opportunity for a competitive hybrid system that can improve significantly the heat supply/storage/remoaval cycle. The challenging question is: how to disseminate these enhancer capsules to increase the effective conductivity without altering the high storage heat capacity of the TES? The global optimizaton strategy starts by the identification of the pertinent parameters, those having the most important influence on the responses of the supply/removal process of the thermal energy in/from the heat composite support ( heat capacity, conductivity, contact resistance ... ) Some parameters are fixed input, others are the output of the optimization process, in particular those that provide the geometrical structure of hybrid media ( radius, thicknes, position of capsules, ...)

### Motion analysis: optimal transport point of view

Zakaria Belhechmi (University of Haute Alsace)

Motion analysis in image analysis consists of modeling the movement in say a video sequence. A central question in this domain is the optical flow estimation, that is a vector field defining the trajectories of elements of a given image to another. The widely dominating model is a pixel-wise transport of the intensity map based on the conservation of the intensity along the trajectories. This approach leads to an ill-posed problem and requires a regularisation procedure. When there are illumination changes, this conservation is no longer valid, and the simplest model consists of adding a new variable to the vector field. In this talk, we discuss a new approach based on optimal mass transportation which copes with the ill-posedness and gives a satisfactory 'explanation" to the very notion of illumination changes.

## Improved error estimates of the splitting methods for the long time dynamics for the weakly nonlinear Schrödinger equations

Yongyong Cai (Beijing Normal University)

We establish improved uniform error bounds for the time-splitting methods for the long-time dynamics of the nonlinear Schrödinger equation (NLSE) with weak nonlinearity. By a new technique of regularity compensation oscillation (RCO) in which the high frequency modes are controlled by regularity and the low frequency modes are analyzed by phase cancellation, an improved uniform error bound is established. Numerical results are reported to validate our error estimates and to demonstrate that they are sharp.

## Parallel Scalable Domain Decomposition Methods for Personalized Biomechanics Simulations and Clinical Applications

Rongliang Chen (Shenzhen Institutes of Advanced Technology, CAS)

The increasing demand for personalized healthcare solutions has brought biomechanical simulations to the forefront of medical diagnostics and treatment planning. Fast and accurate simulations are critical for clinical applications, where timely decision-making can significantly impact treatment outcomes. Biomechanical models, however, are inherently complex, involving multiphysics such as fluid-structure interaction (FSI) and structure-electrical interaction. These models are highly nonlinear, and the corresponding discretized systems are often large and ill-conditioned, posing significant challenges for numerical solution techniques. In this talk, we present some parallel scalable domain decomposition methods for patient-specific biomechanics simulations. These methods enable the efficient and accurate solution of large-scale computational models that capture the complex dynamics of biological systems. By significantly improving computational efficiency and scalability, our approach makes these simulations feasible for real-world clinical applications in personalized medicine. We will also discuss applications of this methodology in cardiovascular simulations, where patient-specific data are used to predict surgical outcomes and optimize treatment strategies, demonstrating the potential for improving patient care.

## Use of Clinical Data for the Constuction of Simplified Personalized FE Models in the Context of Pressure Ulcer Prevention

Carla Cornillon (I2M laboratory, Bordeaux University)

Pressure ulcers are chronic wounds caused by repetitive external loading which can cause internal soft tissue death. The prevention and treatment of these injuries remain a challenge in healthcare. The use of finite element models has increased in research as a means of estimating the internal deformation occurring during these injuries to quantify the underlying injury mechanisms. However, these models remain sensitive to the input material parameters used to model the soft tissue, which motivates the development of methods for which to measure personalized muscle and adipose properties. Additionally, individuals at-risk for pressure ulcers have reported tissue differences from younger healthy individuals, which could influence the conclusions drawn from these. Thus, the goal of this study was to evaluate the soft tissue material differences between different populations through a combined experimental-numerical protocol which utilizes the compressive indentation response of the participant in an inverse optimization procedure to achieve personalized material properties.

### Phase-field method for multi-component vesicle in fluids

Zhenlin Guo (Beijing Computational Science Research Center)

In this presentation, We introduce a thermodynamically consistent phase-field model for multi-component vesicles in fluids. Our model encompassed dual two-phase fluid systems, one to depict the interaction dynamics between the vesicle and its ambient fluid, and the other to capture the dynamics of the multicomponent on the vesicle membrane. These two systems are couped within the diffuse domain framework, which provides a high accuracy approach for solving partial differential equations on complex surfaces. We illustrate a series of numerical examples showcasing classical scenarios and so on. Additionally, we validate the accuracy of our model by comparing these simulations with experiment encountered in multicomponent vesicles, including phase separation, tank-treading and so on. Additionally, we validate the accuracy of our model by comparing these simulations with experimental observations.

## Blow-up result for a weakly coupled system of two euler-poisson-darboux-tricomi equations with time derivative nonlinearity

Mohamed Fahmi Ben Hassen (Imam Abdulrahman Bin Faisal University)

We study the blow-up of solutions to a system of two semilinear wave equations coupled via the timederivative nonlinearities and characterized by linear damping terms in the scale-invariant regime, mass terms, and Tricomi terms. The Tricomi terms are particularly significant from both physical and mathematical perspectives, as they enable time-dependent propagation speeds. Under the assumption that both waves are propagating with the same speed, we derive a new blow-up region for the system under consideration, and we provide a lifespan estimate of the maximal existence time.

## Calculation of the localisation landscape of the stationary waves and of the level of the percolation with FeeFem++

Frédéric Hecht (Sorbonne University)

We want to have a numerical approach to understand the mechanism of Anderson based on the work of Marcel Filoche and Svitlana Mayborodac (see https://www.pnas.org/doi/full/10.1073/pnas.1120432109?doi=10.1073). Let a random medium (the exterior in cube of a given number of spheres with or without overlap distributed with Poisson law). To compute the landscape with we have to solve the solution u of the Poisson equations with unitary right hand side and a correct boundary condition (periodic). The landscape will be 1/u and we want to quickly compute the level  $\lambda$  of percolation of 1/u.( i.e. find the value  $\lambda$  such that a connected component of  $1/u < \lambda$  touches two opposite boundaries of the code)

- We present an algorithm to compute 1/u,
- Find a connected component of a graph (algorithm Kruskal see

https://en.wikipedia.org/wiki/Kruskal0 salgorithm)

- Discretely compute the critical point of 1/u, because  $\lambda$  will be a critical point.
- Results

## A projection based solver for subsonic compressible flows, applications to natural convection problems

Jerome Jansen (Bordeaux INP)

We report a novel method for the treatment of pressure-velocity coupling in the context of single-phase compressible flows. This method can be seen as a generalisation of the Incremental Pressure Correction Method for incompressible flows. The proposed algorithm solves the coupled Navier-Stokes and energy equations written in primitive form plus an Equation of State (EoS). Using Manufactured Solution, time and space convergence has been analysed. We illustrate the ability of achieving second-order temporal accuracy for velocity, pressure, density and temperature fields. Applications to natural convection problems, beyond the Boussinesq approximation, with significant fluid property variations are discussed.

### A Parallel Exponential Integrator Scheme for Linear Differential Equation

Sidi-Mahmoud Kaber (Sorbonne University)

New approximations of the matrix phi-unctions are developed. These approximations are rational functions of a specific form allowing simple and accurate schemes for linear systems. Furthermore, these approximations are fully parallelizable. Several tests show the efficiency of the method and its good parallelization properties.

## A class of higher-order length preserving and energy decreasing IMEX schemes for the Landau-Lifshitz equation

Xiaoli Li (Shandong University)

In this talk we present new higher-order implicit-explicit (IMEX) schemes for the Landau-Lifshitz equation. These schemes are linear, length preserving and only require solving one elliptic equation with constant coefficients at each time step. We show that numerical solutions of these schemes are uniformly bounded without any restriction on the time step size, and establish rigorous error estimates of orders 1 to 5 in a unified framework. Finally numerical examples are presented to validate the proposed schemes.

### Lattice Boltzmann for modeling fluid-structure interaction

Erwan Liberge (La Rochelle University)

The Lattice Boltzmann Method (LBM) has been used in fluid mechanics since the 90s, and his interest has increased with the rise of graphics processing units (GPUs). Indeed, its algorithm, which consists of local computations on mesh nodes and information exchanges on neighboring nodes, is particularly well-suited to parallel computing architectures, especially GPUs, and offers significant gains in terms of computation time. This talk will present 2 methods for modeling Fluid-Structure Interaction using LBM. The first, in the case of rigid solid displacements, consists in penalizing the solid domain in the Lattice Boltzmann method, and thus using LBM for both the fluid and the solid. The second method, in the case of incompressible neo-Hokean solids, involves using a multiphase formulation of the fluid-structure interaction problem.

## Discrete Fracture Matrix Approach based with Petrov-Galerkin IFE for fractured porous media flow

### Hongxing Rui (Shandong University)

The numerical discretization of fluid flow in fractured porous media is extensively employed in, for instance, oil reservoirs, groundwater resource management, and nuclear waste disposal. The simulation and prediction of fluid flow hold immense significance.

In this talk, we propose a numerical discretization for the hybrid-dimensional model, where the information of low-dimensional fractures was embedded into the local functions of interface elements, inspired by IFE methods. Numerical examples, including well-known benchmarks, are performed.

#### Stable higher-order decoupled schemes for Navier-stokes equations

Jie Shen (Eastern Institute of Technology, Ningbo)

Incompressible Navier-stokes equations (NSEs) and their numerical approximations play an important role in many fields of science and engineering. In particular, The NSEs can be coupled with other nonlinear equations, such as Cahn-Hillard, Maxwell, Keller-Segel equations etc, to model various complex phenomenas in fluid mechanics and materials science.

However, how to construct stable higher-order fully decoupled schemes for the NSEs has been a long standing open problem. In this lecture, we shall discuss the main difficulties and review existing approaches in designing higher-order decoupled schemes for NSEs, and present our recent work on a new class of stiffly stable IMEX schemes and apply them to construct unconditionally stable higher-order fully decoupled schemes for the NSEs.

We shall also discuss issues related to the spatial discretizations and present fully discrete decoupled schemes using spectral-Galerkin methods and discontinuous Galerkin finite-elment methods.

## Efficient and Structure-Preserving Algorithms for Gradient Flow Formulation of the Kohn-Sham Density Functional Theory

Shuyu Sun (King Abdullah University of Science and Technology)

The Kohn-Sham density functional theory (DFT) is a commonly used framework for studying many-electron systems in quantum chemistry and physics, especially for calculating molecules' ground state energy and electron configuration. The self-consistent field (SCF) method is typically used to solve these equations, but the SCF iterations may suffer from convergence issues, especially for those open-shell molecules. In this talk, we present efficient and structure-preserving numerical schemes for calculating the electronic structure via the Kohn-Sham density functional theory (DFT) and gradient flow models. The efficiency relies on the numerical scheme's unconditional energy stability, orbital-wise splitting design, and linearity in its implicit part. The structure-preserving property refers both stability and to energy orthonormality-preserving. We unconditionally first give an iterative, structure-preserving, orbital-wise splitting numerical scheme for a first-order gradient flow model with an extended gradient. The splitting ends up with a small system of one orbital at each time step, which can greatly reduce the computational costs when facing large molecules. We then introduce our work on the second-order gradient flow models, which can be regarded as the acceleration of the first-order ones and potentially help escape from the local minimums. We propose a second-order gradient flow model with Lagrange multipliers for the Kohn-Sham DFT and construct а linear,

orthonormality-preserving, orbital-wise splitting mid-point scheme, which can perfectly inherit the energy decay rate of the continuous level. Various numerical examples are performed to verify the theoretical analysis and show the schemes' effectiveness and robustness.

### A Journey into Bound-Preserving Schemes and Theory

Kailiang Wu (Southern University of Science and Technology)

Solutions to many partial differential equations (PDEs) are subject to certain bounds or constraints. For instance, in fluid dynamics, density and pressure must remain positive, while in relativistic cases, fluid velocity must not exceed the speed of light. Developing bound-preserving numerical methods that uphold these intrinsic constraints is crucial. Recently, significant attention has been given to design provably bound-preserving schemes, though challenges remain, particularly for systems with nonlinear constraints.

In this talk, I will present our recent efforts in developing fundamental bound-preserving theories:

1. Geometric Quasilinearization (GQL): Drawing on key insights from geometry, we propose a novel and general framework called geometric quasilinearization. GQL offers an effective approach for addressing bound-preserving problems with nonlinear constraints by transforming these constraints into linear ones through the introduction of auxiliary variables. We establish the fundamental principles and general theory of GQL using the geometric properties of convex regions and present three effective methods for constructing GQL.

2. Optimal Cell Average Decomposition (OCAD): Utilizing convex geometry and symmetric group theory, we develop the optimal cell average decomposition theory, which provides a foundation for constructing more efficient bound-preserving schemes. We demonstrate that the classic Zhang-Shu CAD is optimal in one dimension but generally not in multiple dimensions, thereby addressing their conjecture proposed in 2010.

We apply the GQL and OCAD approaches to various PDEs, showcasing their effectiveness and advantages through diverse and challenging examples and applications, including magnetohydrodynamics (MHD), relativistic hydrodynamics, and the

ten-moment Gaussian closure system.

## An energy-stable mixed finite element method for Rosensweig ferrofluid flow model

### Xiaoping Xie (Sichuan University)

We develop a mixed finite element method for the Rosensweig's ferrofluid flow model. First, we establish some regularity results for the weak solution. Next, for the spatial semi-discretization of the model using mixed finite elements we show that energy inequality of the continuous equation is preserved and give optimal error estimates in  $L^{\infty}(L^2)$  and  $L^2(H^1)$  norms. For the full discretization using implicit Euler scheme we show the existence and uniqueness of solutions, the unconditional stability and optimal error estimates. Finally, we provide numerical experiments to verify the theoretical results.

### Numerical solutions of wave scattering problems: BIEM,FEM and Coupling

Liwei Xu (University of Electronic Science and Technology of China)

Such wave equations as acoustic, elastic and electromagnetic wave equations are the three most common wave equations encountered in physics. Scattering problems associated with these equations have many applications in sciences and engineering. Due to the inherit features of wave propagations, the boundary integral equation method(BIEM) is one of the most desirable computational methods for obtaining numerical solutions of the wave scattering problems. However, as the wave fields coupled with some other physical fields are considered, or as the wave propagates in some complex medium, the discontinuous /continuous finite element method (FEM) will display its advantages for the numerical solutions. In addition, waves usually propagate in an unbounded area, and therefore the coupling of finite and boundary element methods are desired for the numerical solution of scattering problems under such cases. The speaker will present some results on applying these numerical methods solving the wave equations.

## Multidomain Model for Optic Nerve Microcirculation: Roles of Glial Cells and Perivascular Spaces

Shixin Xu (Duke Kunshan University)

The accumulation of potassium in the extracellular space surrounding nerve cells is a fundamental aspect of biophysics that has garnered significant attention in recent research. This phenomenon holds implications for various neurological conditions, including spreading depression, migraine, certain types of epilepsy, and potentially, learning processes. A quantitative analysis is essential for understanding the dynamics of potassium clearance following a series of action potentials. This clearance process involves multiple structures along the nerve, including glia, the extracellular space, axons, and the perivascular space, necessitating a spatially distributed systems approach akin to the cable equations of nerve physiology. In this study, we propose a multi-domain model for the optic nerve to investigate potassium accumulation and clearance dynamics. The model accounts for the convection, diffusion, and electrical migration of fluid and ions, revealing the significant roles of glia and the perivascular space in potassium buffering. Specifically, our findings suggest that potassium clearance primarily occurs through convective flow within the syncytia of glia, driven by osmotic pressure differences. Additionally, the perivascular space serves as a crucial pathway for potassium buffering and fluid circulation, further contributing to the overall clearance process. Importantly, our model's adaptability allows for its application to diverse structures with distinct channel and transporter distributions across the six compartments, extending its utility beyond the optic nerve.

### Applications of the Onsager variational principle in numerical analysis

Xianmin Xu (Academy of Mathematics and Systems Science, CAS)

The Onsager principle is a fundamental law for irreversible processes in statistic physics. It has been used to develop theoretical models for many problems in soft matter. Recently, the variational principle has been used as an approximation tool to derive reduced models for many complicated systems. In this talk, we will present some recent progress on applications the Onsager variational principle in numerical analysis. We will show that the variational formulation makes it convenient to design numerical schemes and to do numerical analysis for dissipative systems.

### A post-processing method for \$Q\$-tensor dynamics preserving the physical range

Jiang Yang (Southern University of Science and Technology)

We propose a post-processing method for \$Q\$-tensor dynamics, which constrains the eigenvalues of \$Q\$ within the physical range \$(-1/3,2/3)\$ by incorporating projection into the time stepping. We show that the projection keeps the eigenframe invariant, from which we derive the explicit formula for the projection that enables easy implementations. Provided that the exact solution lies within the physical range, the projection ensures that the error does not increase. As a result, when the projection is built in a class of stable single-step linearly semi-implicit schemes as post-processing, we establish the error estimates, illustrated by a gradient flow keeping the physical range of \$Q\$. In numerical examples, we examine some defect dynamics for nematic liquid crystals, and the performance of post-processing schemes together with adaptive time-stepping specially designed with projection.

## Perfectly matched layer method for the wave scattering problem by a step-like surface

Weiying Zheng (Academy of Mathematics and Systems Science, CAS)

This talk is concerned with the convergence theory of perfectly matched layer (PML) method for wave scattering problems in a half plane bounded by a step-like surface. When a plane wave impinges upon the surface, the scattered waves compose of an outgoing radiative field and two known parts. The first part consists of two parallel reflected plane waves of different phases, which propagate in two different subregions separated by a half-line parallel to the wave direction. The second part stands for an outgoing corner-scattering field which is discontinuous and represented by a double-layer potential. A piecewise circular PML is defined by introducing two types of complex coordinates transformations in the two subregions, respectively. A PML variational problem is proposed to approximate the scattered waves. The exponential convergence of the PML solution is established by two results based on the technique of Cagniard-de Hoop (CDH) transform. First, we show that the discontinuous corner-scattering field decays exponentially in the PML. Second, we show that the transparent boundary condition (TBC) defined by the PML is an exponentially small

perturbation of the original TBC defined by the radiation condition. Numerical examples validate the theory and demonstrate the effectiveness of the proposed PML.

### Derivative-free deep learning method for parabolic PDEs and HJB equations

Tao Zhou (Academy of Mathematics and Systems Science, CAS)

We propose an efficient derivative-free version of a martingale neural network SOC-MartNet for solving high-dimensional Hamilton-Jacobi-Bellman (HJB) equations and stochastic optimal control problems (SOCPs) with controls on both drift and volatility. This method eliminates the reliance on automatic differentiation for computing temporal and spatial derivatives, offering significant efficiency in solving high-dimensional HJB equations and SOCPs.

## **Participants List**

No.	Name	Affiliation
1	Fouzia Achchaq	Bordeaux University
2	Cyril Alléry	La Rochelle University
3	Mejdi Azaïez	Bordeaux INP
4	Eya Bejaoui	UT Compiègne
5	Faker Ben Belgacem	UT Compiègne
6	Zakaria Belhechmi	University of Haute Alsace
7	Yongyong Cai	Beijing Normal University
8	Rongliang Chen	Shenzhen Institutes of Advanced Technology, CAS
9	Carla Cornillon	I2M laboratory, Bordeaux University
10	Zhenlin Guo	Beijing Computational Science Research Center
11	Mohamed Fahmi Ben Hassen	Imam Abdulrahman Bin Faisal University
12	Frédéric Hecht	Sorbonne University
13	Jerome Jansen	Bordeaux INP
14	Sidi-Mahmoud Kaber	Sorbonne University

15	Xiaoli Li	Shandong University
16	Erwan Liberge	La Rochelle University
17	Zhiping Mao	Eastern Institute of Technology, Ningbo
18	Hongxing Rui	Shandong University
19	Jie Shen	Eastern Institute of Technology, Ningbo
20	Shuyu Sun	King Abdullah University of Science and Technology
21	Kailiang Wu	Southern University of Science and Technology
22	Xiaoping Xie	Sichuan University
23	Tao Xiong	University of Science and Technology of China
24	Liwei Xu	University of Electronic Science and Technology of China
25	Shixin Xu	Duke Kunshan University
26	Xianmin Xu	Academy of Mathematics and Systems Science, CAS
27	Jiang Yang	Southern University of Science and Technology
28	Weiying Zheng	Academy of Mathematics and Systems Science, CAS
29	Tao Zhou	Academy of Mathematics and Systems Science, CAS
30	Zhengjian Bai	Xiamen University
31	Juan Cao	Xiamen University

32	Hongtao Chen	Xiamen University
33	Huangxin Chen	Xiamen University
34	Kui Du	Xiamen University
35	Can Huang	Xiamen University
36	Wen Huang	Xiamen University
37	Xueying Huang	Xiamen University
38	Yumin Lin	Xiamen University
39	Qingxia Liu	Xiamen University
40	Jianxian Qiu	Xiamen University
41	Zhijian Rong	Xiamen University
42	Zhiyu Tan	Xiamen University
43	Congmin Wu	Xiamen University
44	Chuanju Xu	Xiamen University
45	Zhuang Zhao	Xiamen University

## Transportation

## **Routemap of Hotel, Restaurant and Conference Venue**



## Notebook





## **Tianyuan Mathematical Center in Southeast China**

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